



# The Test of Mediation

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This module deals with the study of mediating mechanisms through the analysis of indirect effects.

An appropriate general citation for this material is

Grace, J.B. (2006) *Structural Equation Modeling and Natural Systems*. Cambridge University Press.

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Source: <https://www.usgs.gov/centers/wetland-and-aquatic-research-center/science/quantitative-analysis-using-structural-equation>

**Situation: Post-Fire Recovery of Plant Communities  
in California Shrublands\***

Analysis focus: understand post-fire recovery of  
plant species richness



\*Five year study of wildfires in Southern California in 1993. 90  
plots (20 x 50m), (data from Jon Keeley et al.)



I illustrate the test of mediation using data from an example study that looked at post-fire vegetation recovery in southern California woodlands (actually shrublands, including chaparral).

Citation for that work is:

Grace, J.B. and Keeley, J.E. 2006. A structural equation model analysis of postfire plant diversity in California shrublands. *Ecological Applications* 16:503-514

## Measures and Samples

measured:

- vegetation cover
- species richness
- age of stand that burned
- fire severity

Examination of woody remains allowed for estimate of age of stand that burned as well as severity of the fires.



Following fires, 90 plots were established 20x50m.

A number of measures were taken, as indicated on the slide.

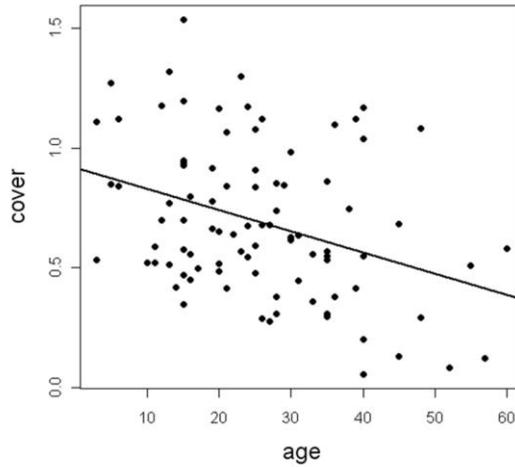


Other factors measured included:

- local abiotic conditions (aspect, soils)
- spatial heterogeneity
- landscape-level conditions (location, elevation)

Additional conditions were measured with an interest in understanding variations in community recovery.

Observation: Post-fire Cover Declines with Age of Stand that Burned

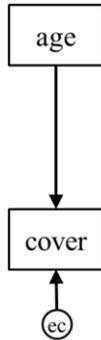


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A key observation was a negative relation between the age of a stand before it burned and the cover of vegetation after the fire.

Lavaan code for evaluating net effect.



```
##### TEST OF MEDIATION #####  
# Net (total) effect of age on cover  
  
mod.1 <- 'cover ~ age'  
  
# Fit the model  
  
mod.1.fit <- sem(mod.2, data=k.dat)  
  
# Extract results  
  
summary(mod.1.fit, stand=T, rsq=T)
```



We can turn that bivariate observation into a net-effects model as shown here.

Lavaan results.

Minimum Function Chi-square	0.000
Degrees of freedom	0
P-value	1.000

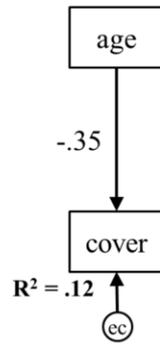
  

	Est	Std.err	Z-value	P(> z )	Std.all
Regressions:					
cover ~					
age	-0.009	0.002	-3.549	0.000	-0.350
Variances:					
cover	0.087	0.013			0.877
R-Square:					
cover	0.123				



Results indicate a significant effect.

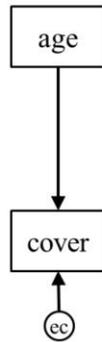
Graphical summary of net relationship.



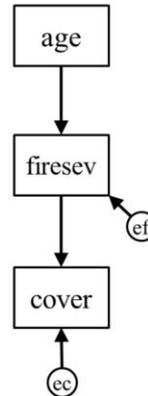
Here is a graphical summary of the net effect.

## The Test of Mediation.

What mediates the causal effect of age on cover?



Could it be that older stands have more severe fires?



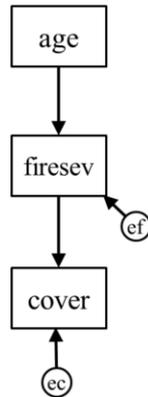
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Now, when I asked Jon Keeley why we might see this relationship, he suggested that older stands would have more fuel and as a result burn hotter (have greater fire severity). More severe fires, in turn, could explain the reduced recovery in older stands. Since he had made measurements of fire severity, we could test that hypothesis formally.

There are different degrees of mediation.

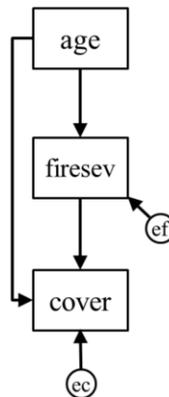
Some Possible Outcomes.

complete mediation



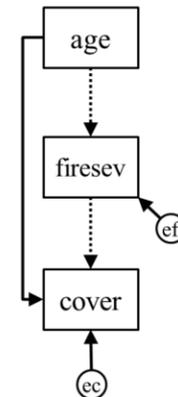
```
'cover ~ firesev
firesev ~ age'
```

partial mediation



```
'cover ~ firesev + age
firesev ~ age'
```

no mediation



```
'firesev ~ 0*age
cover ~ age + 0*firesev'
```

When we think about the possible findings in a test of mediation, there are three types of models possible.

Complete mediation – fire severity can completely explain the influence of stand age.

Partial mediation – fire severity only explains part of the effect of stand age. That would mean some other process was operating as well.

No mediation – of course it could be that observed fire severity did not explain the association between age and cover. For this outcome, either or both of the dashed arrows could be ns = “no mediation”

Note the lavaan code is shown below the models. For the no mediation model I chose to use a lavaan syntax option where the link is included in the model but the parameter is set to zero for the test.

## Use ANOVA function to compare models

```
> anova(comp.mod.fit, partial.mod.fit, nomed.mod.fit)
Chi Square Difference Test
```

	Df	AIC	BIC	Chisq	Chisqdiff	Df	diff
Pr(>Chisq)							
partial.mod.fit	0	1069.4	1081.9	0.0000			
comp.mod.fit	1	1070.7	1080.7	3.2974	3.2974	1	0.069
nomed.mod.fit	2	1096.7	1104.2	31.3526	28.0552	1	1.2e-07

AIC difference and log-likelihood tests both indicate complete mediation model not inferior to partial mediation model. This tilts the decision towards the complete mediation model, which has 1 fewer parameters.



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The anova function performs a likelihood ratio test. We also get the AIC values. All indications are the complete mediation model is an adequate explanation of the data.

We can use AICc to compare the the models.

```
aictab.lavaan(list(comp.mod.fit, partial.mod.fit,  
nomed.mod.fit), c("Complete", "Partial", "None"))
```

Model selection based on AICc :

	K	AICc	Delta_AICc	AICcWt	Cum.Wt	LL
Partial	5	1069.66	0.00	0.64	0.64	-529.69
Complete	4	1070.82	1.16	0.36	1.00	-531.34
None	3	1096.78	27.12	0.00	1.00	-545.37

Results support conclusion that partial and complete models are indistinguishable (Delta\_AICc is less than 2.0). Since the complete mediation model has 1 fewer parameter, I would give it the nod.



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We can go further and create an AICc table, including the computation of model weights. You can refer to the module on “Model Evaluation” for more detail on this procedure.

A succinct treatment of model comparison using AIC tables can be found at

<http://www.unc.edu/courses/2006spring/ecol/145/001/docs/lectures/lecture17.htm>

AICc leads to same conclusions as AIC.

Calculating the magnitude of the standardized indirect effect.



Standardized total effect of age on cover:  
 $= 0.45 \times -0.44 = -.20$



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Simple to compute the indirect effect in the linear Gaussian case, just multiply the path coefficients along the path.

For more complex models, we might use queries to quantify indirect effects.

You can get the intercepts using the “meanstructure” option.

```
# a small digression: asking for the intercepts
partial.mod.fit <- sem(mod.3, meanstructure=T,
  data=k3.dat)
summary(mod.3a.fit)
```

requesting intercepts

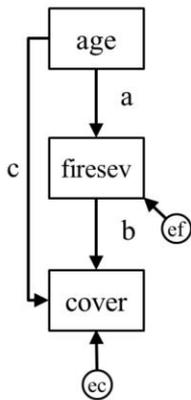
	Est.	Std.err	Z-value	P(> z )
<b>Regressions:</b>				
cover ~				
firesev	-0.839	0.182	-4.611	0.000
firesev ~				
age	0.597	0.124	4.832	0.000
<b>Intercepts:</b>				
cover	10.744	0.883	12.166	0.000
firesev	3.039	0.351	8.647	0.000
<b>Variances:</b>				
cover	8.050	1.200		
firesev	2.144	0.320		



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For prediction equations you will need the intercepts, which require the use of an additional piece of syntax.

We can compute indirect and total effects within lavaan



```
### Compute indirect and total effects
### We will use partial mediation model
mod.4 <- 'cover ~ b*firesev + c*age
         firesev ~ a*age

         direct := c
         indirect := a*b
         total := c + (a*b)

# Fit the model
mod.4.fit <- sem(mod.4, data=k.dat)

# Extract results
summary(mod.4.fit, stand=T, rsq=T)
```



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Here we see that if we label the parameters, we can then define different quantities in the model syntax.

## Results

		Estimate	Std.err	Z-value	P(> z )	Std.all
<b>Regressions:</b>						
cover ~						
firesev	(b)	-0.067	0.020	-3.353	0.001	-0.350
age	(c)	-0.005	0.003	-1.833	0.067	-0.191
firesev ~						
age	(a)	0.060	0.012	4.832	0.000	0.454
<b>Variances:</b>						
cover		0.078	0.012			0.780
firesev		2.144	0.320			0.794
<b>Defined parameters:</b>						
direct		-0.005	0.003	-1.833	0.067	-0.191
indirect		-0.004	0.001	-2.755	0.006	-0.159
total		-0.009	0.002	-3.549	0.000	-0.350
<b>R-Square:</b>						
cover		0.220				
firesev		0.206				

Note that these results will be slightly different from those for the full mediation model.



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Now, we get full information about defined quantities. Here we can see that if you add the direct and indirect effect, you get the total effect.