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Modeling Changes over Time: Time-trajectory Models (aka Growth Models)

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When we have lots of measurements over time, we may wish to generalize things and study trajectories. Now, instead of time steps, we are studying trends and the factors that influence them.

A citation for this work is

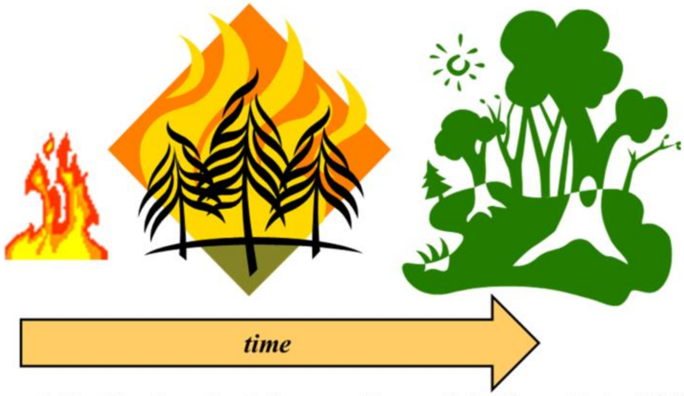
Grace, J.B., Keeley, J., Johnson, D., and Bollen, K.A. 2012. Structural equation modeling and the analysis of long-term monitoring data. pp 325-358. In: Gitzen, R.A., Millspaugh, J.J., Cooper, A.B., and Licht, D.S. Design and Analysis of Long-Term Ecological Monitoring Studies. Cambridge University Press.

Notes: IP-056512; Support provided by USGS Climate & Land Use R&D and Ecosystems Programs. Formal review of this material by Jesse Miller and Phil Hahn. The use of trade names is for descriptive purposes only and does not imply endorsement by the U.S. Government.

Last revised 17.02.26.

Source: https://www.usgs.gov/centers/wetland-and-aquatic-research-center/science/quantitative-analysis-using-structural-equation

Post-fire dynamics recovery (Grace et al. 2012).



Grace, J.B., Keeley, J., Johnson, D., and Bollen, K.A. 2012. Structural equation modeling and the analysis of long-term monitoring data. pp 325-358. In: Gitzen, R.A., Millspaugh, J.J., Cooper, A.B., and Licht, D.S. *Design and Analysis of Long-Term Ecological Monitoring Studies*. Cambridge University Press.

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The study used in this illustration examines the dynamics of post-fire recovery in California shrublands. The hypothesis being examined is that fire rejuvenates diversity of plants in the ecosystem and that following fire, there is a general decline in diversity until the next fire.

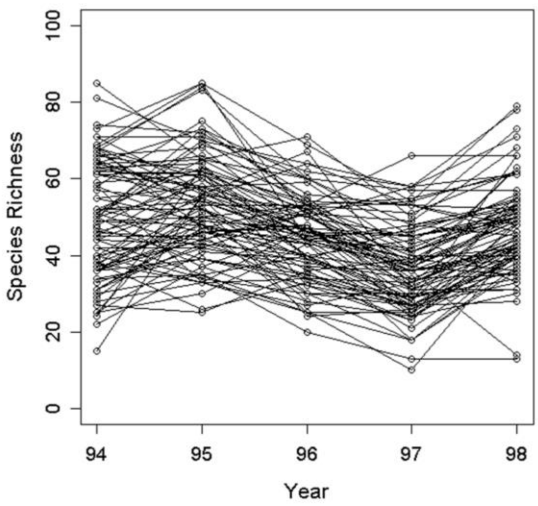

Figure 5. Observed values of herb species richness for the 88 plots in the dataset being examined.

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Diversity dynamics did show sort of a general decline, but with loads of plot-to-plot variation in quantity and pattern. Also, the second and fifth years showed strong upturns, raising questions as to whether there really is a trend as expected.

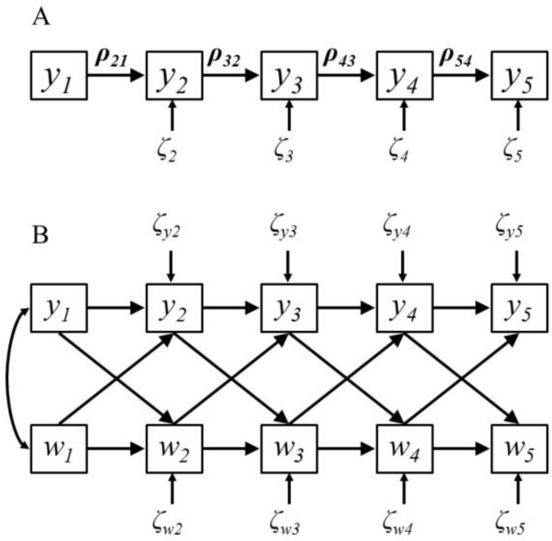

Figure 2. Two examples of autoregressive models. (A) A simple autoregressive chain and (B) an autoregressive cross-lagged model involving a response *y* and a covariate *w*.

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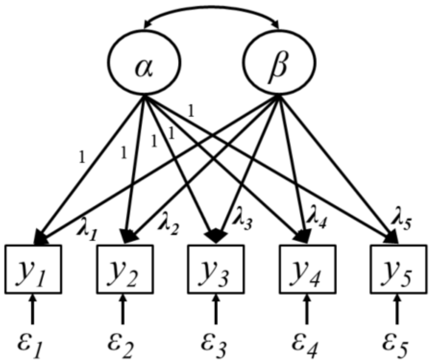
Temporal data are often analyzed as either an autoregressive change or a cross-lag autoregressive model.

Latent Variable Loading Matrix



$$\mathbf{\Lambda} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ \vdots & \vdots \\ 1 & T-1 \end{pmatrix}.$$

Figure 3. Simple latent trajectory model (LTM). In this model the trajectory described by observed measurements of response variable y over 5 time periods can be explained by an intercept α and slope β. For the linear model, the values for $\lambda_1 - \lambda_5 = 0, 1, 2, 3,$ and 4.

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The SEM covariance approach to the problem of temporal dynamics often relies on using latent variables to represent latent slopes and intercepts. There is a need to set intercepts to 1.0 and random slopes are used to set a progression of time steps.

General References:

Bollen, K. A. and P. J. Curran. 2006. Latent curve models: a structural equation perspective. John Wiley & Sons, NY

Duncan, T. E., S. C. Duncan, and L. A. Strycker. 2006. An introduction to latent variable growth curve modeling. 2nd Edition. Lawrence Erlbaum Associates Publishers, Mahwah, NY

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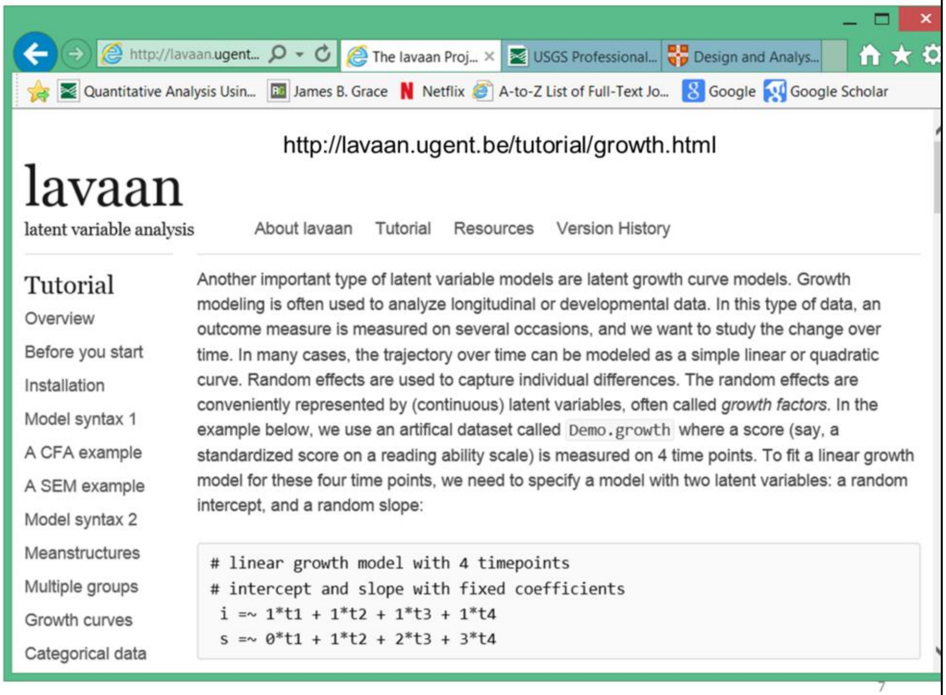
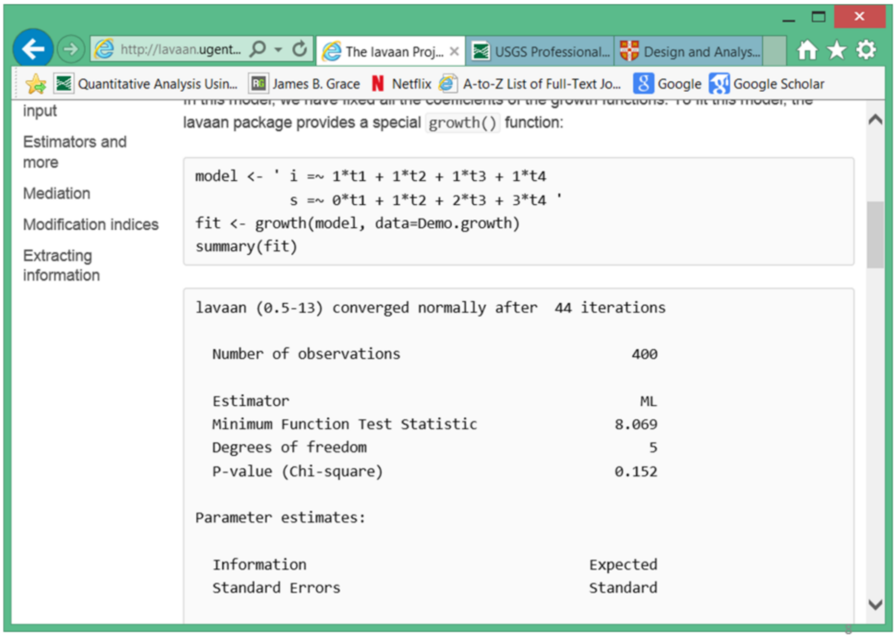
There are now several major references for this model type.



lavaan implements a special function for such models called "growth". He has a tutorial on his training page.




```
model <- ' i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
           s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4 '
fit <- growth(model, data=Demo.growth)
summary(fit)
```

```
lavaan (0.5-13) converged normally after  44 iterations

  Number of observations                           400

  Estimator                                         ML
  Minimum Function Test Statistic                8.069
  Degrees of freedom                                 5
  P-value (Chi-square)                           0.152

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard
```

Screenshot from Rosseel's tutorial.

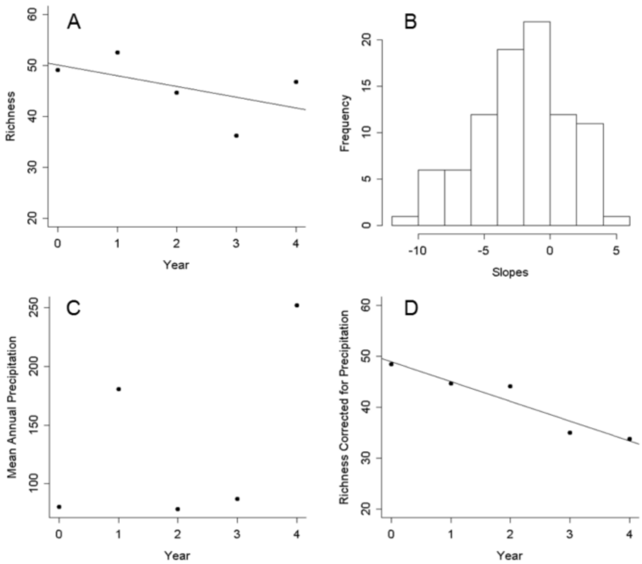


Figure 6. Some characteristics of the data being modeled. (A) Mean richness over time, (B) histogram of individual slopes for the 88 trajectories, (C) mean annual precipitation values, and (D) plot of mean richness corrected for mean annual precipitation.

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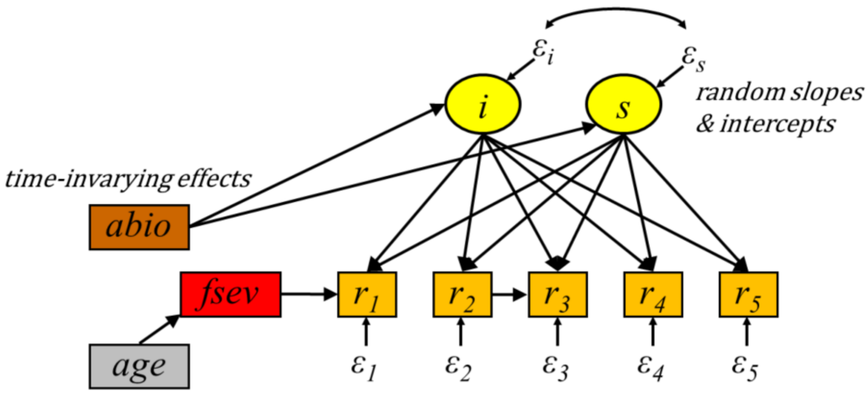
Now, back to our ecological example. Here are some summary statistics.

Hypothesized Latent Trajectory Model for Richness over Time



This is our model after adjusting for precipitation variation.

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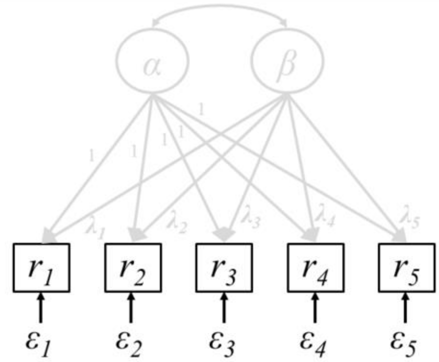
This is a preview of the model we will develop in the subsequent pages. Note there is a good bit of machinery associated with this model type.

Simple linear trajectory

```
### Model 101: simple latent curve
mod.101 <- '
# intercept and slope with fixed coefficients
i =~ 1*r1 +1*r2 +1*r3 +1*r4 +1*r5
s =~ 0*r1 +1*r2 +2*r3 +3*r4 +4*r5'
fit.101 <- growth(mod.101, data=dat2)
```



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We start with the simplest model we can develop for the five time steps. Here the model represents the hypothesis that there is a trend over time. Note that random intercepts apply to each of the time steps (set to 1 in the command statement). A linear slope of change over time is set with the progression of 0, 1, 2, 3, and 4.

Simple linear trajectory model fit

```
> print(fit.101)
lavaan (0.5-20) converged normally after 100
iterations

  Number of observations                            88

  Estimator                                         ML
  Minimum Function Test Statistic               50.303
  Degrees of freedom                                10
  P-value (Chi-square)                           0.000
```

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Model fit statistics show the model does converge, but has poor fit to the data.

Simple linear trajectory results structure

```
Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  i =~
    r1                1.000
    r2                1.000
    r3                1.000
    r4                1.000
    r5                1.000
  s =~
    r1                0.000
    r2                1.000
    r3                2.000
    r4                3.000
    r5                4.000

Covariances:
                   Estimate  Std.Err  Z-value  P(>|z|)
  i ~~
    s                -9.684    5.291   -1.830    0.067
```

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This slide and the next show results.

Simple linear trajectory results structure (cont.)

```
Intercepts:
                   Estimate  Std.Err  Z-value  P(>|z|)
    r1                0.000
    r2                0.000
    r3                0.000
    r4                0.000
    r5                0.000
    i                43.953    1.386   31.708    0.000
    s                -3.991    0.321  -12.430    0.000

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    r1              135.838   24.450    5.556    0.000
    r2               36.455    9.018    4.042    0.000
    r3               63.882   11.439    5.585    0.000
    r4               39.726    8.180    4.857    0.000
    r5               54.884   12.420    4.419    0.000
    i               126.143   26.208    4.813    0.000
    s                 2.443    1.594    1.533    0.125
```

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Additional results.

Simple linear trajectory modification indices

```
> subset(modindices(fit.101), mi > 3.8)
   lhs op rhs     mi     epc sepc.lv sepc.all sepc.nox
3    i =~  r3 11.453   0.070   0.789    0.062    0.062
4    i =~  r4 11.749  -0.064  -0.724   -0.064   -0.064
8    s =~  r3 11.257  -0.771  -1.206   -0.095   -0.095
9    s =~  r4 11.690   0.716   1.119    0.098    0.098
21  r3 ~1        16.400   3.790   3.790    0.299    0.299
22  r4 ~1        13.248  -3.089  -3.089   -0.271   -0.271
26  r1 ~~  r2  8.256  47.500  47.500    0.243    0.243
27  r1 ~~  r3  5.125 -27.587 -27.587   -0.134   -0.134
```

Field observations suggested a carryover effects from year1 to year2 and from year2 to year 3.
This is not exactly what is suggested by the mod indices, but what we will consider first.

And we can request modification indices in order to see some possible modifications to consider. However, in this case, we follow some initial ideas first.

Simple linear trajectory with autoregressive effects.

```
### Model 102: Include autoregressive effects
mod.102 <- '
# intercept and slope with fixed coefficients
i =~ 1*r1 +1*r2 +1*r3 +1*r4 +1*r5
s =~ 0*r1 +1*r2 +2*r3 +3*r4 +4*r5
# autoregressive effects
r2 ~ r1
r3 ~ r2'

fit.102 <- growth(mod.102, data=dat2)
```

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Autoregressive effects are added to the code.

Simple linear trajectory with autoregressive effects.

```
> print(fit.102)
lavaan (0.5-20) converged normally after  93
iterations

  Number of observations                            88

  Estimator                                         ML
  Minimum Function Test Statistic               34.215
  Degrees of freedom                                 8
  P-value (Chi-square)                           0.000
```

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Model discrepancy dropped from 50.3 to 34.2, a clearly significant improvement.

Model 102: modification indices

```
> subset(modindices(fit.102), mi > 3.8)
   lhs op rhs     mi     epc sepc.lv sepc.all sepc.nox
3    i =~  r3  5.832   0.403   4.192    0.334    0.334
4    i =~  r4  7.200  -0.053  -0.550   -0.050   -0.050
5    i =~  r5  5.476   0.066   0.684    0.057    0.057
8    s =~  r3  4.733  -2.148  -2.758   -0.220   -0.220
9    s =~  r4  8.004   0.619   0.795    0.072    0.072
10   s =~  r5  6.254  -0.783  -1.005   -0.084   -0.084
22  r3 ~1        13.145  11.584  11.584    0.923    0.923
23  r4 ~1         9.009  -2.611  -2.611   -0.236   -0.236
24  r5 ~1         5.707   2.975   2.975    0.247    0.247
27  r1 ~~  r2 14.838  61.378  61.378    0.325    0.325
28  r1 ~~  r3  6.200 -28.577 -28.577   -0.144   -0.144
31  r2 ~~  r3  5.029 -18.318 -18.318   -0.122   -0.122
```

Suggesting an error correlation. Dicey in this case, but worth trying.

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Mod indices suggest an error correlation.

Simple linear trajectory with autoregressive effect and error correlation.

```
### Model 103: Include error correlation
mod.103 <- '
# intercept and slope with fixed coefficients
i =~ 1*r1 +1*r2 +1*r3 +1*r4 +1*r5
s =~ 0*r1 +1*r2 +2*r3 +3*r4 +4*r5
# autoregressive effects
r3 ~ r2
r2 ~ r1
# error correlation
r1 ~~ r2'

fit.103 <- growth(mod.103, data=dat2)
```

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Code for adding the error correlation.

Simple linear trajectory with autoregressive effect.

```
> print(fit.103)
lavaan (0.5-20) converged normally after 105
iterations

  Number of observations                            88

  Estimator                                         ML
  Minimum Function Test Statistic               22.390
  Degrees of freedom                                 7
  P-value (Chi-square)                           0.002

> fitMeasures(fit.103, "gfi")
  gfi
0.986
```

Modification indices do not suggest any reasonable additions to make. So, we accept Model 103 for now. Model fit was not too bad and GFI = 0.986

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Indications are there are still some imperfections in the model. Like other latent variable models, this type is a bold prediction that seeks generality over close fit. GFI suggests that fit is pretty good.

Model 3 results

```
Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  r3 ~
    r2                0.105    0.024    4.403    0.000
  r2 ~
    r1                0.031    0.024    1.311    0.190

Covariances:
                   Estimate  Std.Err  Z-value  P(>|z|)
  r1 ~~
    r2               57.666   19.034    3.030    0.002
  i ~~
    s                12.826    6.081    2.109    0.035

Intercepts:
                   Estimate  Std.Err  Z-value  P(>|z|)
    r1                0.000
    r2                0.000
    r3                0.000
    r4                0.000
    r5                0.000
    i                41.918    1.580   26.532    0.000
    s                -3.651    0.395   -9.249    0.000
```

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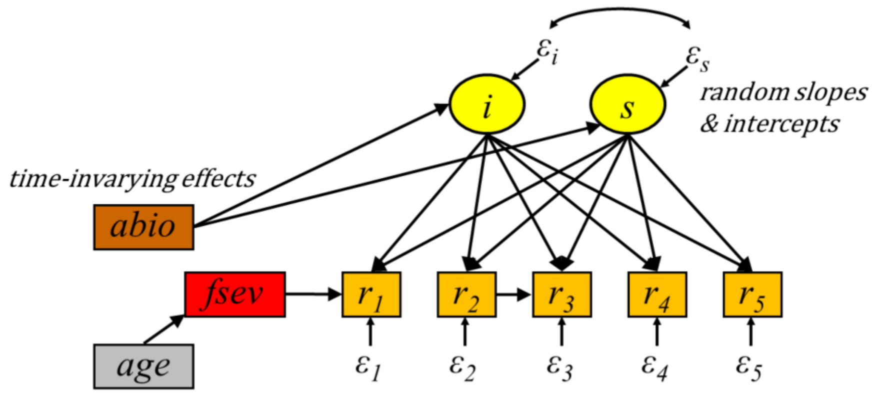
Autoregressive effect from time 2 to 3 is supported, but from time 1 to 2 not supported.

Hypothesized Latent Trajectory Model for Richness over Time



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This figure again shows where we are going, at least in part.

Building out the network of structural effects.

```
### Model 104: Add time-invariant covariates
###            to Model 103
mod.104 <- '
# intercept and slope with fixed coefficients
i =~ 1*r1 +1*r2 +1*r3 +1*r4 +1*r5
s =~ 0*r1 +1*r2 +2*r3 +3*r4 +4*r5
# autoregressive effects
r3 ~ r2
r2 ~ r1
# error correlation
r1 ~~ r2
# time-invariant effects of abiotic
conditions
i ~ abio
# fire severity effects
r1 ~ fire +abio
fire ~ age +abio'
```

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The code here now specifies time invariant effects that can explain the wide variation in intercepts (and means).

Model 4 fit.

```
> fit.104 <- growth(mod.104, data=dat2)
Warning message:
In lav_partable_check(lavpartable, categorical =
categorical, warn = TRUE) :
  lavaan WARNING: missing intercepts are set to zero:
[fire]

> print(fit.104)
lavaan (0.5-20) converged normally after  97
iterations

  Number of observations                            88

  Estimator                                         ML
  Minimum Function Test Statistic               45.603
  Degrees of freedom                                21
  P-value (Chi-square)                           0.001

> fitMeasures(fit.104, "gfi")
  gfi
0.996
```


Non-fatal warning.

Model 4 results

```
Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  r3 ~
    r2                0.156    0.033    4.689    0.000
  r2 ~
    r1                0.135    0.046    2.918    0.004
  i ~
    abio              0.454    0.189    2.397    0.017
  r1 ~
    fire             -2.804    0.697   -4.021    0.000
    abio              0.416    0.090    4.599    0.000
  fire ~
    age               0.073    0.013    5.809    0.000
    abio              0.054    0.007    7.512    0.000
Covariances:
                   Estimate  Std.Err  Z-value  P(>|z|)
  r1 ~~
    r2               13.737   13.086    1.050    0.294
Intercepts:
                   Estimate  Std.Err  Z-value  P(>|z|)
    i                13.599    9.494    1.432    0.152
    s                -1.796    0.824   -2.179    0.029
```

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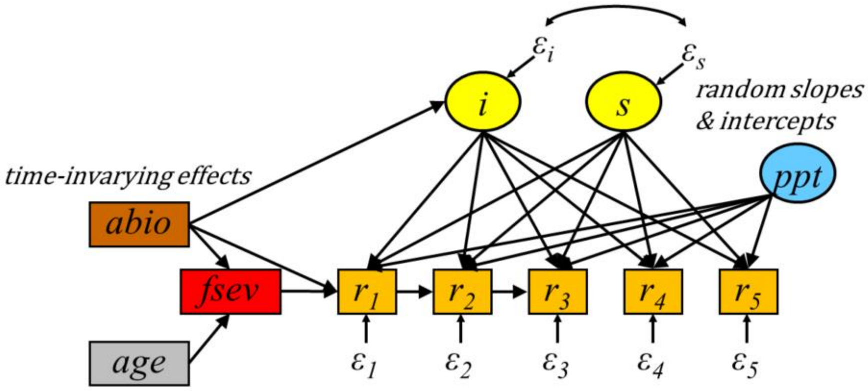
Abiotic favorability effect on the intercept, as well as the other added effects are supported.

Tenative Model for Richness over Time
(showing the adjustment for precipitation as part of the model).



(It may be logical to let the relationship between abio and fire be a correlation instead of directed.)

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This is now the tentative model for richness. Included here, though not shown in the code, is a varying annual precipitation effect that was quite important.