lavaan Syntax Guide[1]

James B. Grace

Contents:

[1] Adapted from Rosseel, Y. The lavaan tutorial (http://lavaan.ugent.be/tutorial/tutorial.pdf) and Beaujean, A.A. Latent Variable Modeling using R. and other sources: Oct. 27, 2019.

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1. Getting Started (return to top)

A few basic points:

Lavaan is an R package for classical, covariance-based structural equation modeling (SEM). To use lavaan, one needs a rudimentary level of familiarity with R; that said, most non-R users can use lavaan syntax with very little knowledge of the R syntax. If you are able to successfully install R on your computer (or get a friend to help), the information in this guide will take you the rest of the way.

An elementary introduction to SEM designed for those in the natural sciences can be found in Grace (2006). Another treatment for biologists with slightly different emphases has been written by Shipley (2016). For first time users in the social sciences, Kline's (2016) book provides an good entry-level treatment.

Links to documentation on lavaan can be found at the lavaan site: http://lavaan.ugent.be/. Included at that site is a more extensive introduction "lavaanIntroduction.pdf" and a technical manual "lavaanIntroduction.pdf".

Lavaan is generally updated fairly frequently, so it is good to keep your version of R up to date. You can download the latest version of R from: http://cran.r-project.org/.

The lavaan package is currently still a beta-version package and not considered complete. That said, it is approaching the functionality of some commercial packages.

The numerical results of the lavaan package are typically very close, if not identical, to the results of the commercial package Mplus. If you wish to compare the results with those obtained by other SEM packages, there are options available for doing so.

In this presentation, as is common in the biometric tradition of structural equations, the inclusion of latent variables in models is considered an advanced topic and covered later.

This presentation focuses on the lavaan command language and does not attempt to provide theoretical background or interpretational information about SEM.

As mentioned above, use of lavaan does not require you to be an expert in R. You will need to know how to import datasets into R and how to execute commands. You also need to know how to install and load packages. The lavaan syntax is simple and requires only general background knowledge, not a deep familiarity with the R language.

a. Loading Data from CSV files: (return to top)

Loading data for use by lavaan is the same as for general R use.

```
## Illustration of loading a .csv file in R
k.dat <- read.csv("lavaan_Syntax_Guide_v2_data.csv")

# Examine Top of Data Set
head(k.dat)

## Load Libraries (assumes libraries already installed)
library(lavaan)
```

b. Creating Data Object from Covariance/Correlation Information: (return to top)

It is possible to convert a covariance or correlation matrix into a data object that lavaan can use for estimate models. While model fitting is illustrated later, here I should the unique syntax for using a covariance matrix with the lavaan "sem" function (sample.cov = name, sample.nobs=n).

```
## Illustration of using a covariance matrix for data
# Create covariance matrix for input
lower <- '
1.0141
0.7177 1.5246
0.3749 0.7686 1.378
'
# Convert information into data covariance matrix
dat.cov <- getCov(lower, names = c("x1", "y1", "y2"))

# When using this data object to fit a model, use the following
fit <- sem(mod, sample.cov = dat.cov, sample.nobs = 100)
```

2. Types of lavaan Commands (return to top)

There are three types of command statements to use when working with lavaan, (a) specification statements, (b) estimation statements, and (c) statements for extracting results. The reader should be aware that there are additional steps in the SEM process, particularly leading from theory to model specification and also following the extraction of results (Grace et al. 2010, Grace et al. 2012).

Here is a preview of the three main lavaan commands, which will be explained subsequently.

```
### Preview of three types of lavaan commands
### Command type 1: specify model by declaring an object
model.1 <- 'y1 ~ x1 + x2       # model defined in quotes
            y2 ~ y1 + x2'

### Command type 2: estimate parameters for the model
model.1.est <- sem(model.1, data=k.dat)  # sem command is used

### Command type 3: extract results from estimated model object
summary(model.1.est, rsq=T)        # summary of results
```

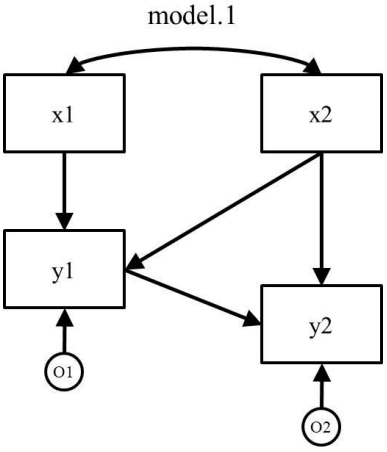
a. Specification of a Model (return to top)

At the heart of the lavaan package is the model 'syntax'. The model syntax is a description of the model to be estimated. In this section, I briefly present the lavaan model syntax for modeling with observed variables. More syntax will be introduced in later sections.

In the R environment, a regression formula has the following form:

y ~ x1 + x2

In lavaan, a typical model is simply a set (or system) of equations contained within quotation marks. Here is a model (model.1) and its syntax:




```
model.1 <- 'y1 ~ x1 + x2
            y2 ~ y1 + x2'
```

Note that the equations (there are two in this example) are "string literals", i.e., by placing them in quotes we make them essentially character statements. Lavaan interprets the statements in their parts, recognizing that there are three variables (y1, x1, and x2) and two operators (~, +) in the first literal and three variables (y2, y1, and x2) and two operators (~, +) in the second as well. Also note exogenous variables are allowed to correlate by default in lavaan.

The basic types of specification operators in lavaan are:

formula type	operator	example
regression	~	y ~ x (y regressed on x)
correlation	~~	y1~~y2 (correlate errors for y1 and y2)
mean/intercept	~ 1	y ~ 1 (estimate mean for y w/o xs)
latent variable definition	=~	BodySize =~ y1 + y2 + y3 (set reflective indicators)
create a composite	<~	Comp1 <~ 1*x1 +x2 +x3 (set formative indicators)
label parameter	*	y ~ b1*x1 + b2*x2 (name coefs. for x1 and x2)
define quantity	:=	TotalEffect := b1*b3 + b2
fix a parameter value	= =	beta1 = = 0
define thresholds	|	u | t1 + t2 (for ordered categorial variable)

Illustrations of latent variable and composite model specification can be found in tutorials on those topics at http://bit.ly/graceSEM. Also refer to Section 6 at the end of this document for pointers to more information related to lavaan syntax.

b. Estimation/Fitting (return to top)

Lavaan has command statements for estimating different types of models. The most basic command is "sem". Here I show how we can estimate the parameters in the above model.

```
model.1.fit <- sem(model.1, data = k.dat)
```

Here, "model.1" refers to the model syntax and "k.dat" is the name for the data object in R.

c. Extracting Results (return to top)

There are a variety of ways of extracting results from the estimated object. You may wish to look at all the main results using a "summary(model.fit)" command. For good modeling practice, one really just wants to see the overall model Goodness of Fit before examining estimated parameters. There are two ways to get fit statistics. The most basic ones can be obtained using

```
show(model.1.fit)
```

```
> show(model.1.fit)
lavaan (0.4-12) converged normally after 40 iterations

  Number of observations                            90

  Estimator                                         ML
  Minimum Function Chi-square                   23.222
  Degrees of freedom                                 1
  P-value                                        0.000
```

Refer to "More Options for Extracting Results" to see how to get more fit indices

The most general extraction statement is the "summary" command, which comes with a variety of options. Again, you can see more details under "More Options for Extracting Results" below.

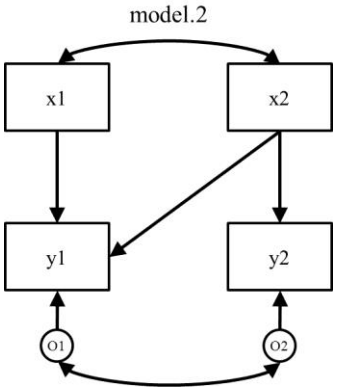
```
summary(model.1.fit)
```

3. More Specification Options (return to top)

There are a number of additional options available that permit further specifications. Here I present several of the basic ones. More advanced commands are presented in a later section.

a. Correlating Errors (return to top)

Let's imagine a case where we have two endogenous responses that have a residual correlation/covariance. In the case where there is residual covariation (literally, a correlation/covariance between the prediction residuals caused by some other unspecified factor influencing both variables), we represent it as an error correlation/covariance.




```
model.2 <- 'y1 ~ x1 + x2
            y2 ~ x2
            y1 ~~ y2'     #error covariance
```

It is important to note here the common convention that when a correlation is specified between two endogenous variables, it is understood that the correlation is a residual correlation and therefore, a correlation between their prediction errors (strictly speaking in causal modeling,

what statisticians would call prediction errors represent other factors affecting a variable). Results presented below include a covariance between y1 and y2, which has now been requested.

```
> summary(model.2.fit)
lavaan (0.4-12) converged normally after 35 iterations

  Number of observations                            90

  Estimator                                         ML
  Minimum Function Chi-square                   22.879
  Degrees of freedom                                 1
  P-value                                        0.000

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
    x1               -0.003    0.004   -0.763    0.446
    x2               -0.087    0.019   -4.643    0.000
  y2 ~
    x2               -3.363    0.896   -3.752    0.000

Covariances:
  y1 ~~
    y2                0.945    0.432    2.189    0.029

Variances:
    y1                0.081    0.012
    y2              195.119   29.087
```

b. Naming Parameters (return to top)

There are a number of operations that require us to name parameters. By naming parameters, we can then specify their values or constrain their values using constraint equations.

Actually, lavaan names parameters automatically using the convention shown in output above. For example, the parameter for the effect of x1 on y1 is named **"`y1 ~ x1`"**.

It can be useful to name parameters in the more conventional way. Since we are used to expressing equations like this,

$$y1 = b1*x1,$$

we might prefer "`b1`" over "`y1 ~ x1`" as a parameter name. The simplest way to do this is to premultiply a predictor with the name being assigned to the parameter. Here we give the parameters in model.2 the names b1-b5. Note, parameter labels must start with a letter!

```
model.2a <- 'y1 ~ b1*x1 + b2*x2
             y2 ~ b3*x2
             y1 ~~ b4*y2'
```

Now, we get the following output, which shows both labels, original and new.

```
> summary(model.2a.fit)
lavaan (0.4-12) converged normally after 35 iterations

  Number of observations                            90

  Estimator                                         ML
  Minimum Function Chi-square                   22.879
  Degrees of freedom                                 1
  P-value                                        0.000

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
    x1       (b1)    -0.003    0.004   -0.763    0.446
    x2       (b2)    -0.087    0.019   -4.643    0.000
  y2 ~
    x2       (b3)    -3.363    0.896   -3.752    0.000

Covariances:
  y1 ~~
    y2       (b4)     0.945    0.432    2.189    0.029

Variances:
    y1                0.081    0.012
    y2              195.119   29.087
```

c. Fixing Parameter Values to Specific Quantities (return to top)

There are times when we want to be able to specify that particular parameters have fixed quantitative values. Lavaan allows us to do this using various options. Here is one approach:

```
model.2b <- 'y1 ~ 0*x1 + x2
             y2 ~ x2
             y1 ~~ y2'
```

In this model statement, x1 is pre-multiplied by zero to set its value to zero. We can also accomplish this using a more elaborate and more flexible approach:

```
model.2c <- 'y1 ~ b1*x1 + x2
             y2 ~ x2
             y1 ~~ y2
             b1 == 0'
```

Now we have labeled the parameter "b1" and then assigned it a value of 0 in a separate statement. This second specification will actually result in an explicit test of the constraint.

```
> summary(model.2c.fit)
lavaan (0.4-12) converged normally after 158 iterations

  Estimator                                         ML
  Minimum Function Chi-square                   23.329
  Degrees of freedom                                 2
  P-value                                        0.000

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
    x1       (b1)     0.000    0.000  169.705    0.000
    x2       (b2)    -0.084    0.018   -4.611    0.000
  y2 ~
    x2       (b3)    -3.362    0.896   -3.752    0.000

Covariances:
  y1 ~~
    y2       (b4)     0.798    0.426    1.872    0.061

Variances:
    y1                0.081    0.012
    y2              195.119   29.087

Constraints:                                 Slack (>=0)
    b1 - 0                                       0.000
```

4. More Estimation Options (return to top)

a. Estimating Intercepts (return to top)

By default, lavaan sets the scales for the variables to zero, placing the emphasis on the other parameters (e.g., path coefficients). One advantage for this default (along with the default of not estimating exogenous covariances) is that we don't estimate as many parameters, which is helpful when sample sizes are limited. However, there certainly are times when we want the estimates for intercepts (e.g., for generating prediction equations). Obtaining these additional parameters is easy, as it only requires overriding a default in the estimation statement. Here we revisit model.1 and ask for intercepts (for endogenous variables) using the "meanstructure" statement.

```
model.1.fit <- sem(model.1, data = data.frame,
+ meanstructure = TRUE)
```

Actually, we could accomplish the same thing by adding command statements of the form "**x1 ~ 1**" to specify means and intercepts as parameters.

```
model.1a <- 'y1 ~ x1 + x2
            y2 ~ y1 + x2
            y1 ~ 1
            y2 ~ 2'
```

Both approaches produce the following results:

```
                   Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
    x1                0.001    0.004    0.327    0.744
    x2               -0.083    0.019   -4.440    0.000
  y2 ~
    y1                9.910    5.083    1.950    0.051
    x2               -2.531    0.976   -2.593    0.010

Intercepts:
    y1                1.004    0.233    4.318    0.000
    y2               53.936    6.925    7.788    0.000

Variances:
    y1                0.080    0.012
    y2              187.212   27.908
```

b. Obtaining Estimates of Correlations/Covariances Between Exogenous Variables (return to top)

Lavaan follows the convention that the exogenous correlations/covariances are not estimated, but instead are taken as pre-estimated in the covariance matrix. This means, if we want to know what the covariances or correlations are between exogenous variables (and we will), we need to obtain them from the data or ask that they be estimated (override the default specification). All we need to do is include an additional statement, "fixed.x=FALSE". Here we return to model.2 and simply ask for the x (exogenous) variables to be freely estimated instead of being fixed at the values found in the covariance matrix (e.g., "fixed.x=FALSE").

```
#estimating the model

model.2d.fit <- sem(model.2, data = k.dat, fixed.x=FALSE)
```

Now we obtain an estimate of the covariance in our lavaan output, as shown in bold below.

```
                   Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
    x1               -0.003    0.004   -0.763    0.446
    x2               -0.087    0.019   -4.643    0.000
  y2 ~
    x2               -3.363    0.896   -3.752    0.000

Covariances:
  y1 ~~
    y2                0.945    0.432    2.189    0.029
  x1 ~~
    x2               -2.651    1.352   -1.961    0.050

Variances:
    y1                0.081    0.012
    y2              195.119   29.087
    x1               58.313    8.693
    x2                2.700    0.402
```

We could have obtained our estimate of the exogenous covariance between x1 and x2 in R simply by using the command "cov()"

```
#estimating covariance between x1 and x2 directly
print(cov(x1,x2))
```

Of course, we can also ask for the full covariance matrix using the following statement.

```
### Ask for the full covariance matrix
print(cov(k.dat))
```

5. More Options for Extracting Results (return to top)

Lavaan has numerous options for obtaining additional output from the model object. Here I focus on 5 key types of information that are commonly required for reporting results and evaluating models.

a. Model Fit Statistics (return to top)

Also of critical importance is the ability to obtain a more complete reporting of model fit statistics. Again, we have two options, one within the "summary" command and another separate function. Again for model.1,

```
summary(model.1.fit, fit.measures=TRUE)
```

yields the following:

```
lavaan (0.4-12) converged normally after 40 iterations

  Number of observations                            90

  Estimator                                         ML
  Minimum Function Chi-square                   23.222
  Degrees of freedom                                 1
  P-value                                        0.000

Chi-square test baseline model:

  Minimum Function Chi-square                   59.220
  Degrees of freedom                                 5
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.590
  Tucker-Lewis Index (TLI)                      -1.049

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -858.441
  Loglikelihood unrestricted model (H1)      -846.830

  Number of free parameters                          6
  Akaike (AIC)                               1728.882
```

```
  Bayesian (BIC)                                1743.881
  Sample-size adjusted Bayesian (BIC)           1724.945

Root Mean Square Error of Approximation:

  RMSEA                                            0.497
  90 Percent Confidence Interval          0.335    0.681
  P-value RMSEA <= 0.05                            0.000

Standardized Root Mean Square Residual:

  SRMR                                             0.134
```

We can also use the extractor function "fitMeasures".

```
fitMeasures(model.1.fit)
```

which produces

```
> fitMeasures(model.1.fit)
           chisq               df           pvalue    baseline.chisq
          23.222            1.000            0.000           59.220
     baseline.df   baseline.pvalue              cfi              tli
           5.000            0.000            0.590           -1.049
            logl unrestricted.logl             npar              aic
        -858.441         -846.830            6.000         1728.882
             bic            ntotal             bic2             rmsea
        1743.881           90.000         1724.945            0.497
  rmsea.ci.lower    rmsea.ci.upper      rmsea.pvalue              srmr
           0.335            0.681            0.000            0.134
```

We can be more surgical with our function and ask for specific measures:

```
fitMeasures(model.1.fit, "bic")
```

which yields

```
> fitMeasures(model.1.fit, "bic")
     bic
1743.881
```

There are also "BIC" and "AIC" functions.

```
> AIC(model.1.fit)
[1] 1728.882
```

```
> BIC(model.1.fit)
[1] 1743.881
>
```

Not all fit measures can be accessed in this way.

b. Modification Indices (return to top)

Diagnosing lack of fit in models is of critical importance. In classical SEM, model fit is evaluated via discrepancies beween observed and model-implied covariances, which are summarized using the above fit measures. Specific discrepancies are also of vital importance. Again, there are two crude approaches.

```
summary(model.1.fit, modindices=TRUE)
```

which yields:

	lhs	op	rhs	mi	epc	sepc.lv	sepc.all	sepc.nox
1	y1	~~	y1	0.000	0.000	0.000	0.000	0.000
2	y1	~~	y2	20.468	-53.694	-53.694	-11.331	-11.331
3	y1	~~	x1	NA	NA	NA	NA	NA
4	y1	~~	x2	0.009	851.155	851.155	1642.024	851.155
5	y2	~~	y2	0.000	0.000	0.000	0.000	0.000
6	y2	~~	x1	20.419	48.615	48.615	0.424	48.615
7	y2	~~	x2	20.468	49.621	49.621	2.010	49.621
8	x1	~~	x1	0.000	0.000	0.000	0.000	0.000
9	x1	~~	x2	0.000	0.000	0.000	0.000	0.000
10	x2	~~	x2	0.000	0.000	0.000	0.000	0.000
11	y1	~	y2	20.468	-0.287	-0.287	-13.657	-13.657
12	y1	~	x1	0.000	0.000	0.000	0.000	0.000
13	y1	~	x2	0.000	0.000	0.000	0.000	0.000
14	y2	~	y1	0.000	0.000	0.000	0.000	0.000
15	y2	~	x1	20.468	0.875	0.875	0.445	0.058
16	y2	~	x2	0.000	0.000	0.000	0.000	0.000
17	x1	~	y1	0.000	0.000	0.000	0.000	0.000
18	x1	~	y2	17.594	0.224	0.224	0.440	0.440
19	x1	~	x2	0.000	0.000	0.000	0.000	0.000
20	x2	~	y1	0.000	0.000	0.000	0.000	0.000
21	x2	~	y2	2.520	0.033	0.033	0.298	0.298
22	x2	~	x1	0.000	0.000	0.000	0.000	0.000

Or we can get the same information using,

```
modindices(model.1.fit)
```

This second approach gives us some flexibility for a more selective options, which are generally preferable. For example, we will typically only want to see modification indices that might be important. For this we can ask to only see "mi" values above, say, 3 (knowing that the cutoff for a critical improvement is an value larger than 3.84).

```
subset(modindices(model.1.fit), mi > 3)
```

We can further extract only those indices that suggest directed arrows be added (i.e., operator is ~).

```
subset(modindices(model.1.fit), mi > 3 & op == "~")
```

(Or only error correlations "~~", which is useful in confirmatory factor models.)

Now we only get the following 4 indices:

```
   lhs op rhs     mi    epc sepc.lv sepc.all sepc.nox
11  y1  ~  y2 20.468 -28.681 -28.681  -13.657  -13.657
12  y2  ~  x1 20.468   0.875   0.875    0.445    5.823
14  x1  ~  y2 16.234   0.206   0.206    0.406    0.406
17  x2  ~  y2 17.106   2.215   2.215    2.025    2.025
```

c. Residual Covariances (return to top)

In addition to looking at modification indices, it can be useful sometimes to look at residuals. Here we are talking about residuals in the covariance matrix, not in the data values themselves (a topic I deal with elsewhere). We can use the "resid" function for this purpose, which includes the option of looking at the standardized residuals.

```
#getting residuals
resid(model.1.fit, type="standardized")
```

which yields,

```
> resid(model.1.fit, type="standardized")
$cov
   y1    y2    x1    x2
y1 0.000
y2 0.000 0.004
x1 0.000 3.942 0.000
x2 0.000 0.000 0.000 0.000
```

Note it may be even better to use type = "cor".

d. Extracting the Parameter Estimates (return to top)

Lavaan has several extraction functions for pulling specific information from the estimated model object. Here I demonstrate one of the most basic, the "parameterEstimates" function. Below I will demonstrate other functions at appropriate places. For model.1, we can extract just the parameter estimates using the following syntax:

```
model.1.fit <- sem(model.1, data = k.dat)
parameterEstimates(model.1.fit)
```

```
> parameterEstimates(model.1.fit)
  lhs op rhs     est     se      z pvalue ci.lower ci.upper
1  y1  ~  x1   0.001  0.004  0.327  0.744   -0.007    0.009
2  y1  ~  x2  -0.083  0.019 -4.440  0.000   -0.119   -0.046
3  y2  ~  y1   9.910  5.083  1.950  0.051   -0.052   19.873
4  y2  ~  x2  -2.531  0.976 -2.593  0.010   -4.444   -0.618
5  y1 ~~  y1   0.080  0.012  6.708  0.000    0.057    0.104
6  y2 ~~  y2 187.212 27.908  6.708  0.000  132.513  241.911
7  x1 ~~  x1  58.314  0.000     NA     NA   58.314   58.314
8  x1 ~~  x2  -2.652  0.000     NA     NA   -2.652   -2.652
9  x2 ~~  x2   2.700  0.000     NA     NA    2.700    2.700
```

Note we get some additional information, the confidence intervals.

The "subset" global function works with many lavaan commands. Here is a simple application that allow you to only view the regression coefs (those lines where the operator is ~ but not ~~).

```
subset(parameterEstimates(model.1.fit), op == "~")
```

This code illustrates how you select any portion of the output.

```
subset(parameterEstimates(modA.fit, standardized=T), op=="~",
c("lhs", "op", "rhs", "std.all"))
```

e. Standardized Estimates (return to top)

We can request standardized coefficients very easily by adding a statement to the summary command. Here we return to model.1 and request standardized parameter estimates and r-squares.

```
summary(model.1.fit, standardized=TRUE, rsq=TRUE)
```

which produces the following (only partial output shown).

```
                   Estimate  Std.err  Z-value  P(>|z|)   Std.lv  Std.all
Regressions:
  y1 ~
    x1                0.001    0.004    0.327    0.744    0.001    0.032
    x2               -0.083    0.019   -4.440    0.000   -0.083   -0.430
  y2 ~
    y1                9.910    5.083    1.950    0.051    9.910    0.208
    x2               -2.531    0.976   -2.593    0.010   -2.531   -0.277

Variances:
    y1                0.080    0.012                     0.080    0.808
    y2              187.212   27.908                   187.212    0.830

R-Square:
    y1                0.192
    y2                0.170
```

Note that the column "Std.lv" only standardizes any latent variables in the model (none in model.1, so that column is same as "Estimate" column). "Std.all" results are what we want in most cases.

f. Model Comparisons (return to top)

We will often wish to compare models using information criteria. Currently, the package "AICcmodavg" supports lavaan objects. Using the command "aictab" we can create a model comparison table.

```
## Compare fun.mod1 and fun.mod1alt
library(AICcmodavg)

aictab(list(fun.mod1.fit, fun.mod1alt.fit),
          c("FunMod1", "FunMod1-Alt"))
```

which produces the following.

```
Model selection based on AICc:

             K   AICc Delta_AICc AICcWt Cum.Wt      LL
FunMod1     33 152.06       0.00    0.6    0.6 -113.15
FunMod1-Alt 32 152.84       0.79    0.4    1.0 -114.82
```

6. Sources of Additional Information

As stated earlier, illustrations of latent variable and composite model specification can be found in tutorials on those topics at http://bit.ly/graceSEM. At that same website one can find numerous advanced topics involving lavaan. Here is a current list of tutorials containing illustrations of the use of lavaan (please note that website is being frequently updated).

1) Introduction to Lavaan (SEM.2.1)
2) Model Evaluation (SEM.3)
3) The Test of Mediation (SEM.5)
4) SEM versus Multiple Regression (SEM.6)
5) SEM versus ANOVA and ANCOVA (SEM.8)
6) All the Modules under "III. Modeling with Latent and Composite Variables"
7) Additional Lavaan Options (SEM.11.1)
8) Multi-Group Modeling (SEM.11.2)
9) Adjusting for Nested Data using lavaan.survey (SEM.11.6)
10) Latent Trajectory Modeling in lavaan (SEM.11.7)

References: (return to top)

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