Reciprocal Effects in Models

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This module deals with the case where two variables affect each other, aka reciprical interactions. The is a type of nonrecursive model.

An appropriate general citation for this material is

Kline, R.B. 2011. Principles and Practice of Structural Equation Modeling. (third edition). Guilford Press, NY.

Note, this is a pre-release copy and not for general distribution.

Last modified: 19.11.19
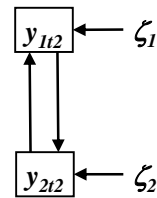
For a real-world SEM application involving reciprocal effects, refer to the paper shown here. This is a rather complex analysis based on a high-dimensional system of variables, so I will not present any of the details here.

The causal diagram (left fig) allows us to treat time in more explicit fashion and can help us understand models with causal loops (right fig).

causal diagram of
reciprocal effects
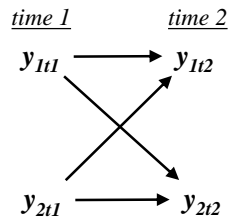
static model
of situation

time 1   *time 2*

y_{1t1} $\longrightarrow$ y_{1t2}

y_{2t1} $\longrightarrow$ y_{2t2}

$\boxed{y_{1t2}} \longleftarrow \zeta_1$

$\boxed{y_{2t2}} \longleftarrow \zeta_2$

The defining causal assumption is that manipulation of y_1 can lead to change in y_2 and vice versa.

What is shown here is that so-called reciprocal effects in models (right figure) are actually what we call "cross-lag" effects when we explicitly consider time (left figure). When we have single-sample data there is a collapse of time in our model (on the right).

There are strong assumptions that have to be made when interpreting SEM results from snap shot data. Interestingly, most of these assumptions have little to do with the causal loop, but spring entirely from the simultaneity of the measurements. The primary assumption is stationarity, which means the relationships among variables are consistent over time.

Houston, we have a problem! – Local non-identification
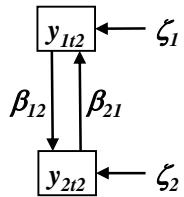
$$S = \begin{bmatrix} var1 & cov12 \\ cov12 & var2 \end{bmatrix}$$

$$\hat{E} = \begin{bmatrix} \zeta_1 & \beta_{21} \\ \beta_{12} & \zeta_2 \end{bmatrix}$$

Observed covariance matrix S only has three unique elements, while the model-implied matrix Ê has four unique parameters.

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Even is we accept the assumption of stationarity and other statistical assumptions regarding errors, we cannot estimate two parameters from a single covariance (i.e., the parameters are not identified). As the next slides will show, this is potentially a solvable problem in SEM.

Inadequate specification.
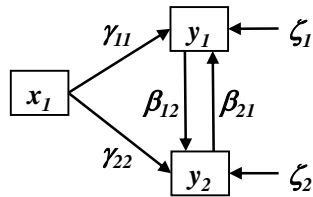
$$y_1 = \gamma_{11}x_1 + \beta_{21}y_2 + \zeta_1$$

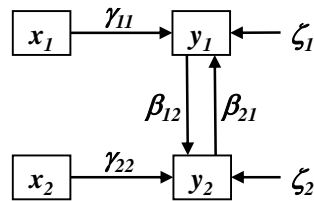nonunique equations

$$y_2 = \gamma_{22}x_1 + \beta_{12}y_1 + \zeta_2$$

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If we bring in a third variable, we now have a 3X3 matrix, which has three off-diagonal covariances. In this case we are still under-identified. and our equations are not unique.

Minimal requirement for full model identification, unique predictors ("instruments") for the reciprocating variables.

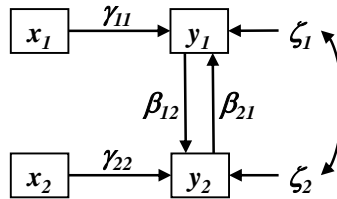$$y_1 = \gamma_{11}x_1 + \beta_{21}y_2 + \zeta_1$$

$$y_2 = \gamma_{22}x_2 + \beta_{12}y_1 + \zeta_2$$

Unique equations

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For identification, it is recognized that a minimum requirement is unique equations for each endogenous variable. This generally means some unique predictor for at least one of the variables involved in the reciprocal effect (a unique predictor for each, as shown here, is even better).

We assume, until we find otherwise, that the errors for the reciprocating variables may be correlated.

$$\boxed{x_1} \xrightarrow{\gamma_{11}} \boxed{y_1} \leftarrow \zeta_1$$

$$\beta_{12} \quad \beta_{21}$$

$$\boxed{x_2} \xrightarrow{\gamma_{22}} \boxed{y_2} \leftarrow \zeta_2$$
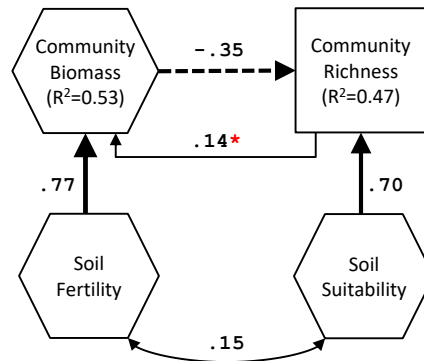

In a paper by

Wong, C.S. and Law, K.S. 1999. Testing reciprocal relations by nonrecursive structural equation models using cross-sectional data. *Organizational Research Methods* 2:69-87.

The authors explore a number of factors that influence the ability to extract proper estimates in such models of these. Among the things they show is that when there are correlated errors among the endogenous variables, as shown here, one should include the error correlation to obtain valid estimates.

California Grassland Example
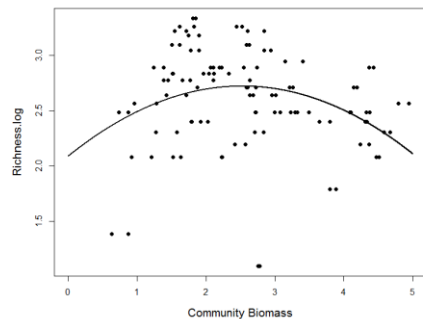
It is traditionally assumed that this pattern fits the "humped-back model", which has no diversity effect on production.

Community Biomass (R^2=0.53) — −.35 → Community Richness (R^2=0.47)

.14*

.77

.70

Soil Fertility

Soil Suitability

.15

There has been a lot of debate over the effects of productivity/biomass on richness and vice versa. There are actually processes that can cause effects in both directions. Here is an example submodel extracted from a study

Grace, James B., Susan Harrison, and Howard Cornell. "Is biotic resistance enhanced by natural variation in diversity?." *Oikos* 126, no. 10 (2017): 1484-1492.

Using the methods described in modules dealing with Composites (and explicitly the methods shown in the Exercise using Two Composites), it is possible to identify the model and separate the effects that are confounded in univariate regression analyses.

In this example, the asterisk shows that the effect of Community Richness on Community Biomass is non-significant in this case.

The model is identified due to the weak correlation between composites for soil fertility and soil suitability.

The ultimate test is whether the model achieves identification. If it does not, you will receive an error message.